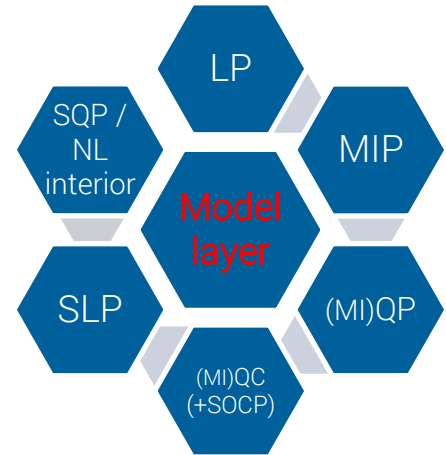
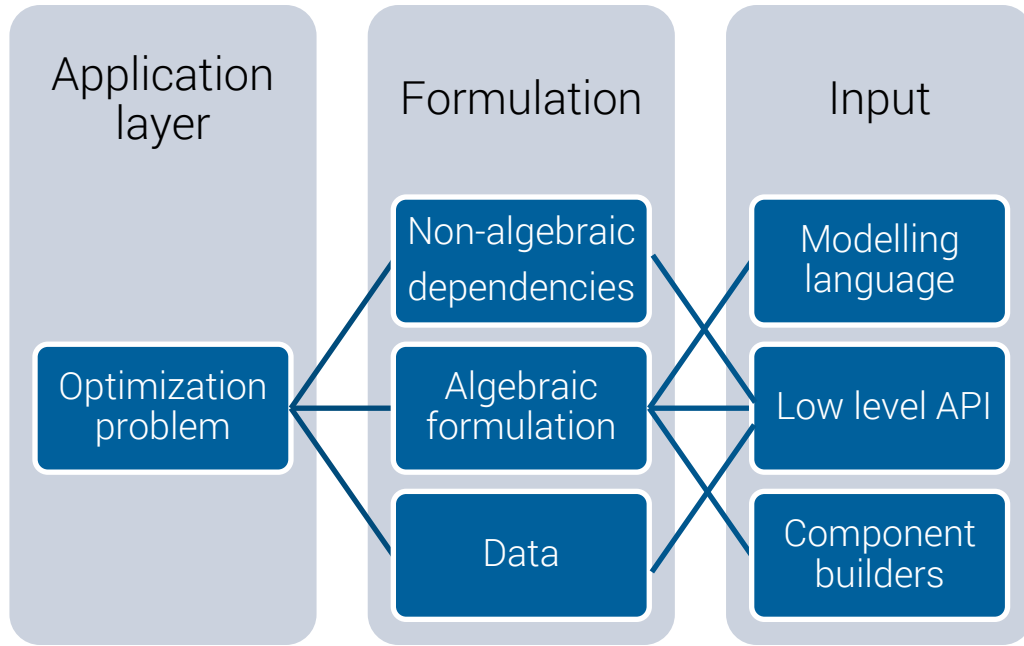




Nonlinear improvements

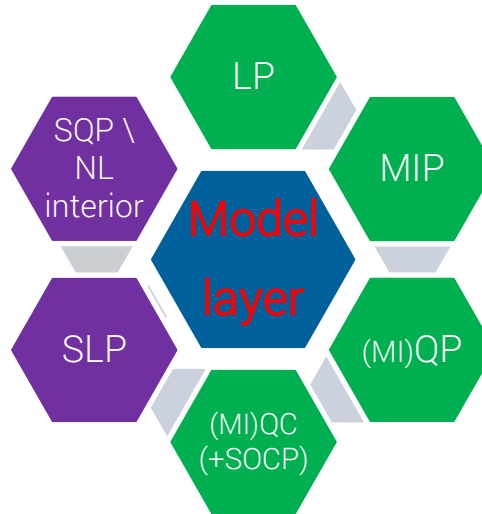
The modelling layer



Nonlinear means a generic solver is necessary

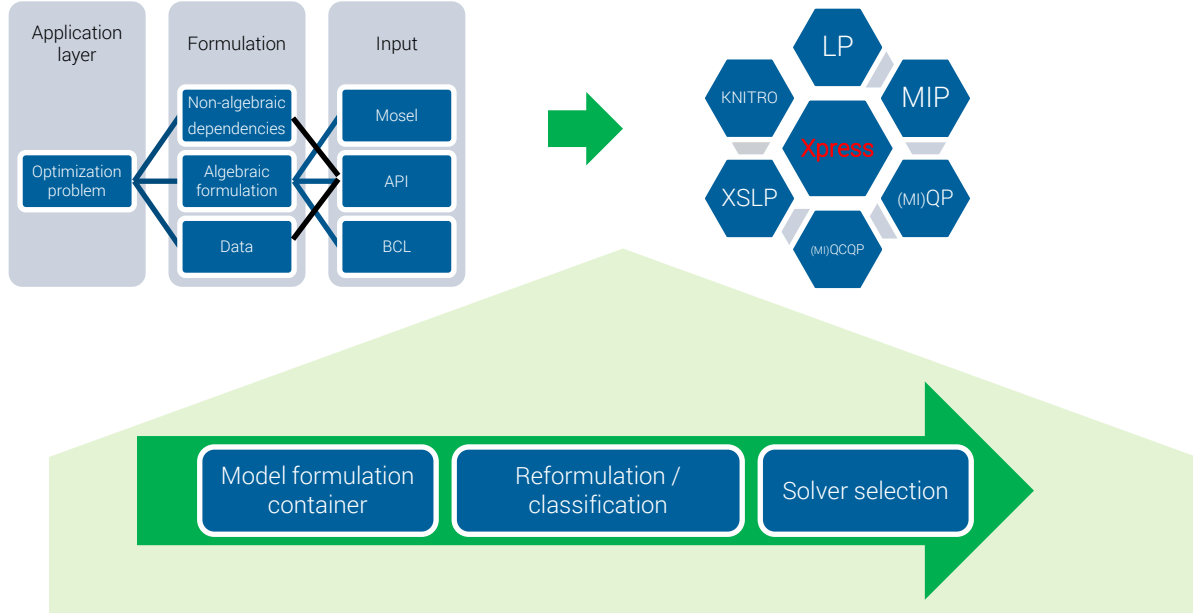
Interior and active
set solvers

Successive linear
programming



Purpose written
solvers

Single entry to all solvers



"I just want to run a simple optimization here. It is probably linear in nature but was leveraging the non-linear code we had for convenience."

Major consulting company, 2015 October

Improved nonlinear bound and domain propagation

More accurate linearizations

Limits freedom in penalty variables for expressions with non-trivial implied bounds

At least one globally optimal solution is retained

Applied in MINLP after a feasible solution is found
(new cutoff -> tighter bounds)

On by default on
XSLP_PRSEOLVEOPS

Benefits:

- Tighter approximations
- Improved problem class recognition (by fixing)
- faster MINLP
- Improves problems with function domain issues

Example: presolve simplifying the problem

propagat Output/Input

```

model "propagate"

uses "mmsnlp"

declarations
x, y, z : mpvar
end-declarations

setparam("xnlp_verbose", 1)

x <= 0
x^2 + (y-2)^4 = log(x+1)
x + z <= 5
y + z >= 3

minimize( x + y + z )

end-model

```

Reading Problem \xprs_9a973f0
Problem Statistics
2 (0 spare) rows
3 (0 spare) structural columns
4 (0 spare) non-zero elements
Global Statistics
0 entities 0 sets 0 set members
Using Knitro [C:\Zsolit\Xpress 7.9 64\bin\xknitro.dll]
Loaded KNITRO 9.1.0
Bound Tightening reduced 2 bounds
Presolve removed 3 fixed variables
Removed 1 constant expressions
Problem is nonlinear presolved
Minimizing problem using Xpress-Optimizer
Minimizing LP \xprs_9a973f0
Original problem has:
3 rows 4 cols 4 elements
Presolved problem has:
0 rows 0 cols 0 elements

Its	Obj Value	S	Ninf	Mneg	Sum Dual Inf	Time
0	3.000000	D	0	0	.000000	0

Uncrunching matrix
Optimal solution found
Dual solved problem
0 simplex iterations in 0s

Final objective : 3.0000000000000000e+00
Max primal violation (abs / rel) : 0.0 / 0.0
Max dual violation (abs / rel) : 0.0 / 0.0
Max complementarity viol. (abs / rel) : 0.0 / 0.0
All values within tolerances

Output/Input | Stats | Matrix | Solutions | Objective | MIP search | BB tree | User graph
IIS

propagat Output/Input

```

model "propagate"

uses "mmsnlp"

declarations
x, y, z : mpvar
end-declarations

setparam("xnlp_verbose", 1)
setparam("xslp_presolve", 0)

x <= 0
x^2 + (y-2)^4 = log(x+1)
x + z <= 5
y + z >= 3

minimize( x + y + z )

end-model

```

Problem Characteristics
Objective goal: Minimize
Number of variables: 4
bounded below: 1
bounded above: 0
bounded below and above: 0
fixed: 3
free: 0
Number of constraints: 3
linear equalities: 0
nonlinear equalities: 1
linear inequalities: 2
nonlinear inequalities: 0
range: 0
Number of nonzeros in Jacobian: 6
Number of nonzeros in Hessian: 2

Iter	Objective	FeasError	OptError	Step	CGI
0	3.990000e+000	0.000e+000			
3	3.000000e+000	0.000e+000	5.944e-010	2.438e-005	

EXIT: Locally optimal solution found.

Final Statistics
Final objective value = 3.000000000059444e+000
Final feasibility error (abs / rel) = 0.00e+000 / 0.00e+000
Final optimality error (abs / rel) = 5.94e-010 / 5.94e-010
of iterations = 3
of CG iterations = 0
of function evaluations = 4
of gradient evaluations = 4
of Hessian evaluations = 3
Total program time (secs) = 0.003 (0.016 CPU
Time spent in evaluations (secs) = 0.000

Output/Input | Stats | Matrix | Solutions | Objective | MIP search | BB tree | User graph
IIS

Example: bound reduction tightening formulation

The image displays two screenshots of the Xpress-SLP solver interface, illustrating the process of bound reduction tightening.

Left Screenshot (Initial Run):

```

model "domain"
uses "mnmnlp"

declarations
x, y, z : mpvar
end-declarations

setparam("xnlp_verbose", 1)
setparam("xslp_solver", 0)

log(x) >= 0.1

minimize( x )

end-model
    
```

Output/Output:

```

Reading Problem \xprs_9a973f0
Problem Statistics
      0 (      0 spare) rows
      1 (      0 spare) structural columns
      0 (      0 spare) non-zero elements
Global Statistics
      0 entities      0 sets      0 set members
Bound Tightening reduced 1 bound
Problem is nonlinear presolved
Maximum expanded nl-formula size: 3
  Jacobian: symbolic differentiation
            1 base AD formula, 4 average complexity
            1 in the Jacobian, 5 average complexity
Minimizing problem using Xpress-SLP
Xpress-SLP Augmentation Statistics:
Columns:
  1 implicit SLP variables
  1 delta vectors
  1 penalty error vectors (1 positive, 0 negative)
Rows:
  1 nonlinear constraints
  1 update rows
  1 penalty error rows
Coefficients:
  3 non-constant coefficients

Itr. LPS  NetObj      ErrorSum      ErrorCost  Unconv. Exten
  1  0    1.258925    .000000    .000000      1
  2  0    1.258925    .000000    .000000      0

Xpress-SLP stopped after 2 iterations. 0 unconverged items
No unconverged values in active constraints
    
```

Right Screenshot (After Tightening):

```

model "domain"
uses "mnmnlp"

declarations
x, y, z : mpvar
end-declarations

setparam("xnlp_verbose", 1)
setparam("xslp_presolveops", 0)
setparam("xslp_solver", 0)

log(x) >= 0.1

minimize( x )

end-model
    
```

Output/Output:

```

Xpress-SLP error: argument 0.000000e+000 out of range in function
104  0      .000000      .000000      .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
105  0    1831.665039    300.081683    157242540.6    1
106  0    915.832520    .000000    .000000      1
107  0    457.916260    .000000    .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
108  0      .000000      .000000      .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
109  0    457.916260    300.095421    204424660.8    1
110  0    228.958130    .000000    .000000      1
111  0    114.479065    .000000    .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
112  0      .000000      .000000      .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
113  0    114.479065    300.098855    265755100.3    1
114  0    57.239532     .000000    .000000      1
115  0    28.619766     .000000    .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
116  0      .000000      .000000      .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
117  0    28.619766     300.099714    345482618.9    1
118  0    14.309883     .000000    .000000      1
119  0     7.154942     .000000    .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
120  0      .000000      .000000      .000000      1
Xpress-SLP error: argument 0.000000e+000 out of range in function
Xpress-SLP error: argument 0.000000e+000 out of range in function
121  0     7.154942     300.099928    449127725.8    1
122  0     3.577471     .000000    .000000      1
123  0     1.788735     .000000    .000000      1
124  0     1.160441     .000000    .000000      1
125  0     1.254969     .000000    .000000      1
126  0     1.258919     .000000    .000000      0

Xpress-SLP stopped after 126 iterations. 0 unconverged items
No unconverged values in active constraints
    
```

New: Catching formula evaluation errors

Helps to identify not well-defined parts of the problem

XSLPsetcbcoeferror \ XSLPprintevalinfo.

Itr.	LPS	NetObj	ErrorSum	ErrorCost	Unconv.	Extended	Action
1	0	-6.000000	.000000	.000000	1	0	
Xpress-SLP error: argument -1.000000e+000 out of range in function SQRT							
-----> SORT (X + 1): 'X = -2.000000'							
Xpress-SLP error: argument -1.000000e+000 out of range in function LN							
-----> LN (Y + 1): 'Y = -2.000000'							
2	0	2.000000	.000000	.000000	0	0	

Xpress-SLP stopped after 2 iterations. 0 unconverged items

No unconverged values in active constraints

Formulas with evaluation errors:

[1] (R4), term: = SQRT (X + 1)

[1] (R5), term: = LN (Y + 1)

Observed error in the log

Runtime analysis

Final summary

Nonlinear variable elimination

Multi-period nonlinear forecasting models:

- Limited number of decision variables

- Large regressions expressions forecasting model parameters for subsequent periods:
repeatedly embedded recursive formulas

Expanded formulas get extensively large

- A recent case: ~100 variables yielding $> 2^{32}$ expression tokens

Solution approaches:

- Transfer variables (harder problems due to artificially constraining the model)

- (virtual) Eliminations, XSLP_PRESOLVEOPS**

- Cascading** (even if the linear expressions have feasible ranges)

Targets **models with a large number of intermediate variables**

Eliminations example

```

Reading Problem \xprs_9a973f0
Problem Statistics
  120 (    0 spare) rows
  640 (    0 spare) structural columns
  240 (    0 spare) non-zero elements

Global Statistics
  0 entities      0 sets      0 set members
Unique objective transfer row __mxxnlp_ObjTransferRow through variable __mxxnlp_ObjTransferVar
Presolve eliminated 128 variables
  eliminated formula size : 239684
  linear substitutions   : 40
  formula substitutions   : 498
Presolve removed 284 fixed variables
Maximum expanded nl-formula size: 142263
Maximizing problem using Xpress-SLP
Xpress-SLP Augmentation Statistics:
  Columns:
    128 delta vectors
  Rows:
    1 nonlinear constraints
    128 update rows
  Coefficients:
    201 non-constant coefficients

Itr. LPS   NetObj      ErrorSum   ErrorCost   Unconv. Extended   Action
  1  0  1225528.274    .000000    .000000     72      0
  2  0  1250879.286    .000000    .000000     27      0
  3  0  1251476.332    .000000    .000000     36      0
  4  0  1260759.857    .000000    .000000     27      0
  5  0  1249428.030    .000000    .000000      0      0

Xpress-SLP stopped after 5 iterations. 0 unconverged items
No unconverged values in active constraints
  
```

```

Reading Problem \xprs_9a973f0
Problem Statistics
  120 (    0 spare) rows
  640 (    0 spare) structural columns
  240 (    0 spare) non-zero elements

Global Statistics
  0 entities      0 sets      0 set members
Unique objective transfer row __mxxnlp_ObjTransferRow through variable __mxxnlp_ObjTransferVar
Presolve removed 284 fixed variables
Problem is nonlinear presolved
Maximum expanded nl-formula size: 14219
Maximizing problem using Xpress-SLP
Xpress-SLP Augmentation Statistics:
  Columns:
    184 delta vectors
    256 penalty error vectors (128 positive, 128 negative)
  Rows:
    129 nonlinear constraints
    184 update rows
    1 penalty error rows
  Coefficients:
    1616 non-constant coefficients

Itr. LPS   NetObj      ErrorSum   ErrorCost   Unconv. Extended   Action
  1  0  1225510.523    .000000    .000000     184     0
  2  0  1250879.286    .000000    .000000     104     0
  3  0  1251476.332    .000000    .000000     120     0
  4  0  1260759.857    .000000    .000000     104     0
  5  0  1249428.030    .000000    .000000     120     0
  6  0  1252971.847    .000000    .000000     104     0
  7  0  1251476.332    .000000    .000000     120     0
  8  0  1260759.857    .000000    .000000     104     0
  9  0  1247283.286    .000000    .000000      88     0
 10  0  1297116.296    520134.6752 -1938870.606   184     0
 11  0  1228880.095    377435.4007 -1687320.560   168     0
 12  0  1132095.385    622529.8638 -4226924.096   184     0
 13  0  1136770.885    299178.1932 -2616608.212   134     0
 14  0  1127104.266    191234.5933 -1979384.878   136     0
 15  0  1165248.543    124853.3356 -1684073.709   151     0
 16  0  1189874.840    62614.79837 -1156080.551   133     0
 17  0  1193790.071    54852.18592 -1136454.674   119     0

...
 51  0  1190367.797     860.344142 -129814934.9    29     0
 52  0  1190320.985    237.545605 -47802609.58    30     0
 53  0  1190354.183     54.464514 -13506033.02     1     2
 54  0  1190372.216     7.299334 -2353102.124     0     0
 55  0  1190374.354     .000000    .000000         0     0

Xpress-SLP stopped after 55 iterations. 0 unconverged items
No unconverged values in active constraints
  
```

New MINLP heuristics

```

Problem Statistics
  972 (    0 spare) rows
 1019 (    0 spare) structural columns
 4603 (    0 spare) non-zero elements
Global Statistics
  14 entities      0 sets      0 set members
  
```

```

...
Itr. LPS   NetObj      ErrorSum      ErrorCost      Unconv. Extended  Action
  1  0      80.269823    23.015655     300.090173     118      0
  2  0     -103.349132  .000000      .000000        83      0
  3  0     -102.862524  .000000      .000000        72      0
  4  0     -102.869118  .000000      .000000         0      0
  
```

xpress-SLP stopped after 4 iterations. 0 unconverged items
 No unconverged values in active constraints

```

Starting heuristics
*  0  1  -93.318014  -102.869118      1      1  10.23%      6      0s
Heuristics stopped
Setting tree search absolute cutoff to -93.318
  
```

} MINLP heuristics

Invoking SLP global search
 MIP strategy: SLP within MIP

```

Node Sols   BestSoln      BoundEstim      Active  Depth   Gap      SLPIts  Time
  0  1  -93.318014  -102.869118      0      1  10.23%      6      0s
  1  1  -93.318014  -102.869118      1      2  10.23%      7      0s
  
```

```

...
  47  1  -93.318014  -94.308525      1      7   1.06%     140     3s
  48  1  -93.318014  -94.222045      0      3   .96%     141     3s
Minimizing problem using Xpress-SLP
  
```

```

Itr. LPS   NetObj      ErrorSum      ErrorCost      Unconv. Extended  Action
  1  0     -93.318014  .000000      .000000         0      0
  
```

xpress-SLP stopped after 1 iterations. 0 unconverged items
 No unconverged values in active constraints

MINLP heuristics integrated into
 the MIP search just as for normal
 MILP
 XSLP_HEURSEARCHSTRATEGY

Presolve affinity and reduced memory footprint

XSLP_PRESOLVE: low memory alternative

when memory is scarce

for one time problems

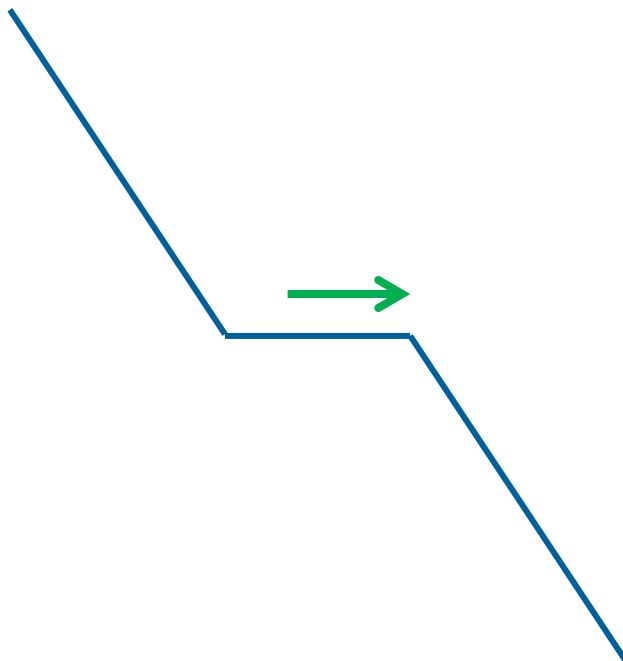
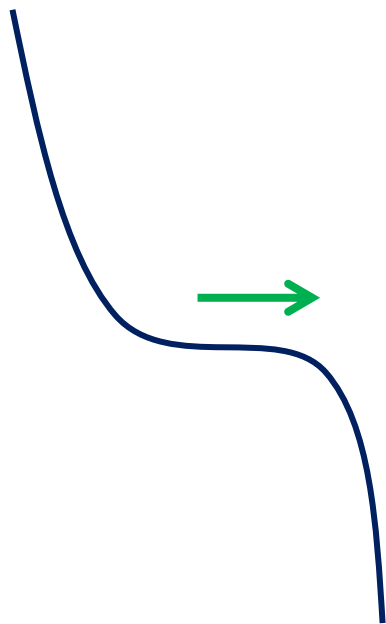
XSLP_PRESOLVELEVEL:

maintain recognisability and high level structures
allow for adjusting model between iterations

Improved partial derivative perturbation logic

XSLP_DELTAZLIMIT: Placeholder management against local optimality \ stalling

Added focus on piecewise linear and ABS expressions

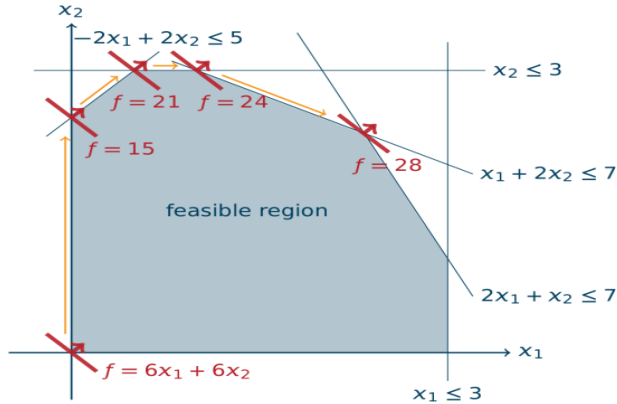


Use higher level information for guessing a good perturbation

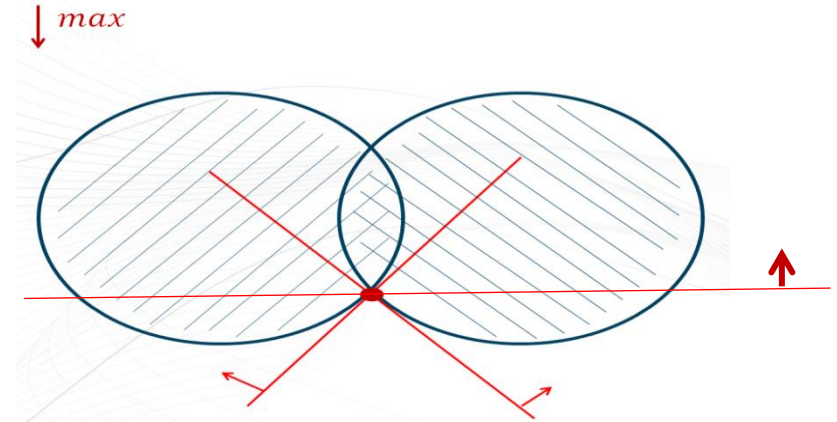


Characterising optimality

Dual multipliers: proof of optimality & importance measure of constraints



Complementarity slackness:
a constraint can only be used to
describe optimality, if the
constraint is active



Example (barrier accuracy)

$$\min x_1^2 + 2x_2^2 - 2x_1x_2 - 2x_1 - 6x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 2 \\ -x_1 + 2x_2 &\leq 2 \\ 2x_1 + x_2 &\leq 3 \end{aligned}$$

Default Barrier behaviour, 6 iterations:

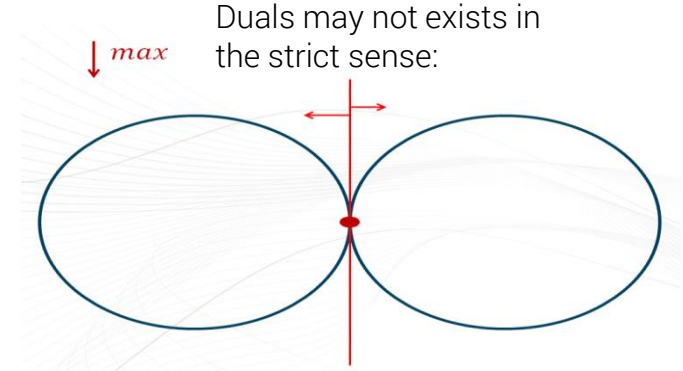
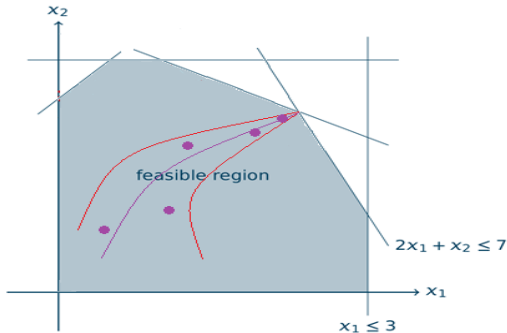
Objective function value is -8.222205

Rows Section

	Number	Row	At	Value	Slack Value	Dual Value	RHS
L	1	C0000001	BS	1.999998	.000000	-3.110855	2.000000
L	2	C0000002	BS	1.999976	2.38437E-05	-.444505	2.000000
L	3	C0000003	BS	2.666671	.333329	-1.59298E-04	3.000000

Columns Section

	Number	Column	At	Value	Input Cost	Reduced Cost
C	4	x1	SB	.666673	-2.000000	1.63725E-05
C	5	x2	SB	1.333325	-6.000000	.000000



Stricter convergence, 8 iterations:

Objective function value is -8.222222

Rows Section

	Number	Row	At	Value	Slack Value	Dual Value	RHS
L	1	C0000001	BS	2.000000	.000000	-3.111111	2.000000
L	2	C0000002	BS	2.000000	.000000	-.444444	2.000000
L	3	C0000003	BS	2.666667	.333333	.000000	3.000000

Columns Section

	Number	Column	At	Value	Input Cost	Reduced Cost
C	4	x1	SB	.666667	-2.000000	.000000
C	5	x2	SB	1.333333	-6.000000	.000000

Improved linearization focused first order optimality checks

	Linearization dual KKT error	Validated KKT	Objective
SLP out of the box	1.128766e-002	8.401041e-003	.657246
SLP tight convergence	8.938231e-005	7.906361e-005	.657164
Strict convergence only	0.000000e+000	0.000000e+000	.657164
Knitro (second order, slower)	0.000000e+000	0.000000e+000	.657164

Objective versus KKT validation error

	BASELINE		FEASTOL		TIGHT		STRICT	
Bental4	-350	1	-350	1	-350	1	-450	0
Adhya4	-372.324	0.997473	-470.833	0.000003	-470.833	0		
gppL5	-372.324	0.997473	-470.833	0.000003	-470.833	0		
Bental5	-900.096	0.957827	-3500	0				
gppL7	-900.096	0.957827	-3500	0				
gppL14	-750	0.666653	-750	0.5	-750	0.5	-750	0
Adhya1	-0.00374	0.438568	-46.364	0.999967	-56.6667	0		
gppL2	-0.00374	0.438568	-46.364	0.999967	-56.6667	0		
gppD5	-1641.12	0.396933	-1641.47	0.022933	-1641.25	0		
gppC4	-1468.64	0.357727	-1494.2	0.001553	-1481.03	0.002318	-1493.95	0.000006
Foulds5	-3.3E-05	0.215883	-6.99874	0.000069				
gppL11	-3.3E-05	0.215883	-6.99874	0.000069				
Adhya3	0.000006	0.200149	0.000005	0.116873	0	0.000017		
gppL4	0.000006	0.200149	0.000005	0.116873	0	0.000017		
gppB4	-912.8	0.139405	-912.8	0				
gppC3	-1706.12	0.097499	-1706.12	0.000002	-1706.12	0		
gppD3	-2069.04	0.078942	-2069.06	0.925463	-2069.06	0		
gppD2	-1180.06	0.053774	-1180.06	0.000011	-1180.06	0		
Adhya2	-549.802	0.00348	-549.803	0.001203	-549.803	0.001203	-549.802	0
gppL3	-549.802	0.00348	-549.803	0.001203	-549.803	0.001203	-549.802	0
gppL15	-4391.83	0.001564	-4391.83	0.000022	-4391.83	0		
gppC5	-1071.81	0.000887	-1071.81	0.000237	-1071.81	0		
gppD4	-626.686	0.000871	-618.518	0				
gppE1	-463.227	0.000049	-463.227	0.000002	-463.227	0		
gppE4	-891.253	0.000003	-891.253	0.000111	-891.253	0		
Foulds2	-600	0						
Foulds3	-8	0						
Foulds4	-7.5	0						

Some other convenience additions

XSLP_STOPSTATUS:

Allows checking how SLP terminated (if a limit was hit)

XSLP_ECHOXPRSMESSAGES:

If the SLP message callback should serve as the XPRS message callback as well

XSLP_MIPSOLS:

Retrieve how many integer solutions were found during the MINLP search
(this may be larger than XPRS_MIPSOLS)

Nonlinear improvement summary

Xpress 7.9

Performance improvements

- Significantly improved nonlinear **bound and domain propagation**
- Variable **elimination**
- New **MINLP heuristics** in the nonlinear B&B search

New functionality

- **Reduced memory footprint** solve path alternative
- Adjustable presolve level

Analysis

- Improved perturbation logic for **degenerate partial derivatives**
- Analysing source of **evaluation errors**

Convenience additions

- XSLP_STOPSTATUS
- XSLP_ECHOXPRSMESSAGES
- XSLP_MIPSOLS